Integralwettbewerb 2025

May 27, 2025

Hochschule Hannover

1. Vorrunde

Vorrunde 1

$$I_1 = \int_0^1 (1+x)e^x dx = e$$

$$I_2 = \int \sin(x)\cos(x) dx = -\frac{1}{4}\cos(2x) + C$$

$$I_3 = \int \frac{1}{x^2+4x+5} dx = \arctan(x+2) + C.$$

$$I_4 = \int x e^x dx = e^x \big(x-1\big) + C$$

$$I_5 = \int \frac{4x}{x^2+1} dx = \ln\sqrt{|3x^4 - 5x^2 + 1|} + C$$

2. Vorrunde

Vorrunde 2

$$I_1 = \int \sin^4 x \cos x dx = \frac{1}{5} \sin^5 x + C$$

$$I_2 = \int e^{2x} \sin^2 x + e^{2x} \cos^2 x dx = \int e^{2x} dx = \frac{e^{2x}}{2} + C'$$

$$I_3 = \int \frac{3}{\cos^2(4x-2)} dx = \frac{3}{4} \tan(4x-2) + C$$

$$I_4 = \int \frac{3e^x + 3 - 2e^x}{e^x + 1} \, \mathrm{d}x = 3x - 2 \ln\left(e^x + 1\right) + C$$

$$I_5 = \int \sin(\sqrt{x}) dx = -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) + C$$

3. Vorrunde

Vorrunde 3

$$I_1 = \int 4x\cos\left(2x^2\right) dx = \sin(u) + C' = \sin\left(2x^2\right) + C$$

$$I_2 = \int x\sin x dx = -x\cos x + \sin x + C.$$

$$I_3 = \int \frac{x^2+4}{x+1} dx = \frac{x^2}{2} - x + 5\ln|x+1| + C$$

$$I_4 = \int \frac{6x^2+4x}{2x^3+2x^2+6}\,\mathrm{d}x = \ln\left|2x^3+2x^2+6\right| + C$$

$$I_5 = \int \sin(2x)\cos(2x) dx = -\frac{1}{2}\cdot\frac{\cos(4x)}{4} + C' = -\frac{\cos(4x)}{8} + C'$$

4. Vorrunde

Vorrunde 4

$$I_1 = \int \frac{x+2}{x^2+4x+5}\,dx = \frac{1}{2}\ln\left(x^2+4x+5\right) + C$$

$$I_2 = \int \frac{x^2+2x+1}{\sqrt{x+1}}\,dx = \frac{2}{5}(x+1)^{5/2} + C.$$

$$I_3 = \int 5\sin(2x-1)\,dx = -\frac{5}{2}\cos(2x-1) + C$$

$$I_4 = \int e^{x+e^x}\,dx = e^{e^x} + C$$

$$I_5 = \int \frac{2x^4 - 3\sqrt{x}}{7\sqrt[3]{x^4}}\,dx = \frac{6}{77}x^{\frac{11}{3}} - \frac{18}{7}x^{\frac{1}{6}} + C$$

Virtelfinale Gruppe A & B

Runde 1

$$I_{\mathrm{l}} = \int \frac{8x^2 - 2x - 43}{(x+2)^2(x-5)} dx$$

$$I_{\mathrm{r}} = \int \cos x \sqrt[3]{3 + \sin x} dx$$

$$I_l = 5 \ln|x+2| - \frac{1}{x+2} + 3 \ln|x-5| + C$$

$$I_r = \frac{3}{4}\sqrt[3]{(3+\sin x)^4} + C$$

Runde 2

$$I_l = \int \frac{dx}{x^2\sqrt{x^2+1}} dx$$

$$I_r = \int \frac{x}{\cos^2 x} dx$$

$$I_l = -\frac{\sqrt{x^2+1}}{x} + C$$

$$I_r = x \tan x + \ln|\cos x| + C$$

Runde 3

$$I_{\mathrm{l}} = \int \frac{x \arcsin\left(x^2\right)}{\sqrt{1-x^4}} dx$$

$$I_{\mathrm{r}} = \int \frac{8x^3 - 20x}{x^4 - 5x^2 + 4} dx$$

$$I_l = \frac{1}{4}\left(\arcsin\left(x^2\right)\right)^2 + C$$

$$I_r = 2\ln\left|x^4 - 5x^2 + 4\right| + C$$

Viertelfinale Gruppe C & D

Runde 1

$$I_{\mathrm{l}} = \int x^2 \cdot (1+x)^{2025} dx$$

$$I_{\mathrm{r}} = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$I_l = \tfrac{1}{2028}(1+x)^{2028} - \tfrac{2}{2027}(1+x)^{2027} + \tfrac{1}{2026}(1+x)^{2026}$$

$$I_r = -\ln|z| + C = -\ln|\sin x + \cos x| + C$$

Runde 2

$$I_{\mathrm{l}} = \int \frac{\ln\left(xe^x\right)}{x} dx$$

$$I_{\mathrm{r}} = \int \frac{x}{\cos^2 x} dx$$

$$I_l = x + \tfrac{1}{2}(\ln x)^2 + C$$
$$I_r = \tfrac{1}{2}(\arctan x)^2 + C$$

Runde 3

$$I_{\mathrm{l}} = \int \frac{1}{x^3+x} dx$$

$$I_{\mathrm{r}} = \int \frac{3e^x+3-2e^x}{e^x+1}\, \mathrm{d}x$$

$$I_l = \ln|x| - \frac{1}{2}\ln\left(x^2+1\right) + C$$
$$I_r = 3x - 2\ln\left(e^x+1\right) + C$$

Halbfinale

Runde 1

$$I_1 = \int \cos^5 x dx$$

$$I = \int (\ln x)^2$$

$$I_l = \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$$

$$I_r = x(\ln x)^2 - 2x\ln x + 2x + C$$

Runde 2

$$I_l = \int_0^1 \frac{16x - 16}{x^4 - 2x^3 + 4x - 4} dx$$

$$\int \frac{\ln^2 x}{x^2}\, \mathrm{d}x$$

$$I_l = \pi$$

$$I_r = -\frac{\ln^2 x + 2\ln x + 2}{x} + C$$

Runde 3

$$I_\mathrm{l} = \int e^{\sqrt{x}} \,\mathrm{d}x$$

$$I_\mathrm{r} = \int x^n \ln x dx$$

$$I_l = 2e^{\sqrt{x}}(\sqrt{x} - 1) + C$$

$$I_r = \frac{x^{n+1}}{n+1}\left(\ln x - \frac{1}{n+1}\right) + C$$

Platz 3

Runde 1

$$I = \int \frac{ln(3x)dx}{x \cdot ln(x)} = \ln 3 \ln |\ln x| + \ln |x| + C,$$

Runde 2

$$I = \int_0^{2025} \lceil x \rceil - \lfloor x \rfloor dx = 2025$$

Floor-Funktion $\lfloor x \rfloor$: Rundet immer ab auf die nächste ganze Zahl
Ceiling-Funktion $\lceil x \rceil$: Rundet immer auf auf die nächste ganze Zahl

Runde 3

$$I = \int \frac{e^x \sinh x}{e^x + 1}\,\mathrm{d}x = \frac{e^x - x}{2} + C$$

Finale

Runde 1

$$I = \int \lim_{n\to\infty} \left(\sum_{i=0}^{n} \left(\frac{1}{x} \right)^{i} \right) dx = x + \ln|x-1| - 1 + C$$

Runde 2

$$I = \int x^3 \sqrt{36 - x^2} dx = -\frac{1}{5} \left(36 - x^2\right)^{\frac{3}{2}} \left(x^2 + 24\right) + C$$

Runde 3

$$I = \int_0^{\frac{\pi}{2}} \frac{e^{-\tan(x)}}{1 - \sin^2(x)} \mathrm{d}x = 1$$